A maintenance model with a quasi generalized Lindley distribution

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Introduction

- Lindley distribution (1958) [3, 4] was derived using the Bayes' theorem and in the last years has been in the attention of statisticians as a suitable model for lifetime data
- Lindley distribution has better properties that the exponential one (2008)[2]
- The hazard rate function of the Lindley distribution is increasing and not constant like the one of the exponential distribution
- There are many generalizations and compounding of the Lindley distribution: Pareto Poisson Lindley (2013) [6], beta exponentiated power Lindley (2015) [7], negative binomial Lindley (2010) [5], quasi Lindley geometric (2014) [1], beta-Lindley (2014) [2], power Lindley (2013) [3].

Aim of paper

- Presentation of a new maintenance shock model using geometric processes
- A example of this model using Lindley type distributions
- Using the mixture of the Lindley distribution, we compare Lindley type distributions with exponential and gamma type distributions
- Simulations for the maintenance shock model

Quasi Lindley distribution [4](2013)

Let $X \sim QGL(a, \theta)$ be a random variable quasi Lindley distributed. Then the probability density of X is

$$f(x) = \frac{\theta}{a+1} e^{-\theta x}(a + \theta x) \qquad (1)$$

and the corresponding cumulative function

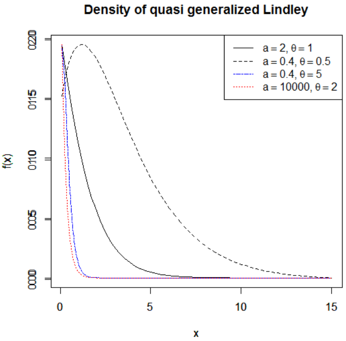
$$F(x) = 1 - \frac{a+1+\theta x}{a+1} e^{-\theta x}, \quad a > 0, \theta > 0 \qquad (2)$$

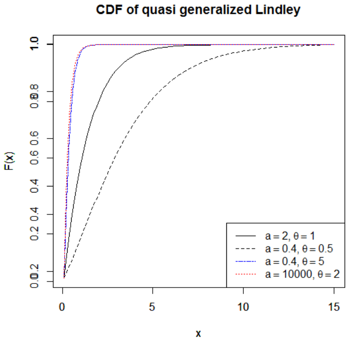
- Mixture of gamma distributions

$$f(x) = pg(x) + (1-p)h(x)$$

where $g(x) = \theta e^{-\theta x}$, $h(x) = \theta^2 x e^{-\theta x}$ and $p = \frac{a}{a+1}$

Quasi Lindley



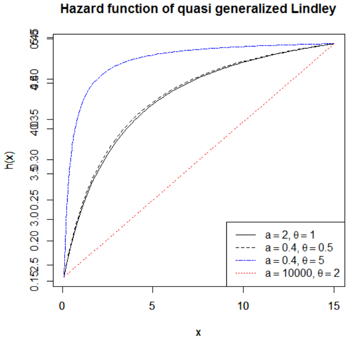


Hazard rate function

The hazard rate function is

$$h(x) = \frac{\theta(a+\theta x)}{a+1+\theta x}$$

h(x) is IFR.



Mean and variance

The mean and the moments of order 2, 3 and 4 are

$$\mu = \frac{a+2}{\theta(a+1)}, \qquad \mu_2 = \frac{2(a+3)}{\theta^2(a+1)}, \qquad \mu_3 = \frac{6(a+4)}{\theta^3(a+1)},$$

$$\mu_4 = \frac{24(a+5)}{\theta^4(a+1)}$$

The variance of X is

$$\sigma^2 = \frac{a^2+4a+2}{\theta^2(a+1)^2}$$

Entropy

Shannon entropy

$$H(f) = -ln\frac{\theta^2}{a+1} + \frac{a+2}{a+1} - \frac{(a+1)(lna - ln\theta) - e^a Ei(-a)}{a+1}$$

where $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t} dt$ is the integral exponential function

Renyi entropy

$$\mathcal{J}_R(\gamma) = \frac{1}{1-\gamma}\left\{2\gamma ln\theta + a\gamma + ln\Gamma(\gamma+1, a\gamma) - \gamma ln(a+1) - (\gamma+1)ln\theta\gamma\right\},$$

$\gamma > 0, \gamma \neq 1$

Generation of date

1. Generate $u_i \sim U(0,1)$
2. Generate $v_i \sim Exp(\theta)$
3. Generate $w_i \sim Gamma(2,\theta)$
4. If $u_i \leq p = \frac{a}{a+1}$ then $x_i = v_i$, otherwise $x_i = w_i$

Transmuted quasi Lindley [4](2013)

Transmuted distributions have been introduced to model data sets in various areas

- engineering-reliability data-transmuted Weibull distribution
- finance, economics, modeling positive data- transmuted Lomax distribution
- biomedical sciences- transmuted inverse exponential distribution

The Rank Transmutation Map (QRTM) is defined as

$$G_{R12}(u) = u + \lambda u(1-u), \quad |\lambda| \leq 1$$

Using cdf's we have the relationship

$$F_2(x) = (1+\lambda)F_1(x) - \lambda F_1(x)^2$$

and for densities

$$f_2(x) = f_1(x)[(1+\lambda) - 2\lambda F_1(x)]$$

where $f_1(x)$ and $f_2(x)$ are the corresponding probability density function associated with the cumulative density functions $F_1(x)$ and $F_2(x)$, respectively. For more details see Shaw et al. (2007)[8].

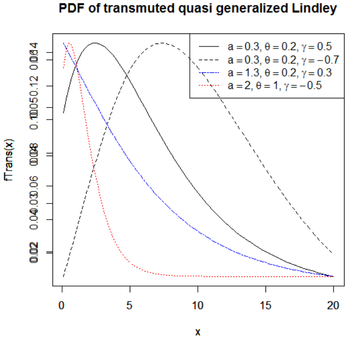
Transmuted quasi Lindley

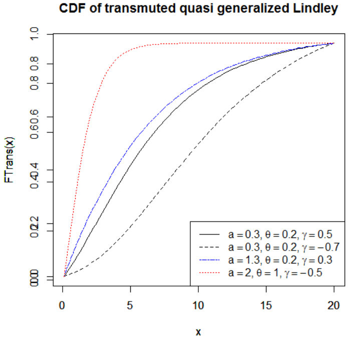
Let $X \sim TQGL(a, \theta, \gamma)$, $a, \theta > 0, |\gamma| \leq 1$. The pdf is

$$f(x) = \frac{\theta}{a+1} e^{-\theta x}(a+\theta x)\left[(1-\gamma) + 2\gamma \frac{a+1+\theta x}{a+1} e^{-\theta x})\right]$$

and the cdf

$$F(x) = \left[1 - \frac{a+1+\theta x}{a+1} e^{-\theta x}\right]\left[1 + \lambda \frac{a+1+\theta x}{a+1} e^{-\theta x}\right], \quad x > 0$$


PDF of transmuted quasi generalized Lindley
a = 0.3, θ = 0.2, γ = 0.5
a = 0.3, θ = 0.2, γ = −0.7
a = 1.3, θ = 0.2, γ = 0.3
a = 2, θ = 1, γ = −0.5
fTrans(x)
0
5
10
15
20
x


CDF of transmuted quasi generalized Lindley
a = 0.3, θ = 0.2, γ = 0.5
a = 0.3, θ = 0.2, γ = −0.7
a = 1.3, θ = 0.2, γ = 0.3
a = 2, θ = 1, γ = −0.5
FTrans(x)
0
5
10
15
20
x

The transmuted quasi Lindley has the following submodels

- $\lambda = 0$ we get the quasi Lindley distribution
- $\lambda = -1$ we get the exponentiated quasi Lindley
- $a = 0$ we get the transmuted gamma distribution
- $a \to \infty$ we get the transmuted exponential distribution

The mean of X is

$$\mu = \frac{(1+\gamma)(a+2)}{8(a+1)} - \frac{2\gamma}{8\theta(a+1)^2}\Big[(a+2)(8\theta-2(a+1))-1\Big]$$

The hazard rate function of X is

$$h(x) = \frac{\frac{\theta}{a+1}e^{-\theta x}(a+\theta x)\left[(1-\gamma)+2\gamma\frac{a+1+\theta x}{a+1}e^{-\theta x}\right]}{1-\left[1-\frac{a+1+\theta x}{a+1}e^{-\theta x}\right]\left[1+\gamma\frac{a+1+\theta x}{a+1}e^{-\theta x}\right]}$$

Data generation

1. Generate $u_i \sim Uniform(0,1)$
2. Generate $v_i \sim Exp(\theta)$
3. Generate $w_i \sim Gamma(2,\theta)$
4. If $u_i < p = \frac{a}{a+1}$ $x_i = (1+\gamma)v_i - \gamma v_i^2$ otherwise $x_i = (1+\gamma)w_i - \gamma w_i^2$

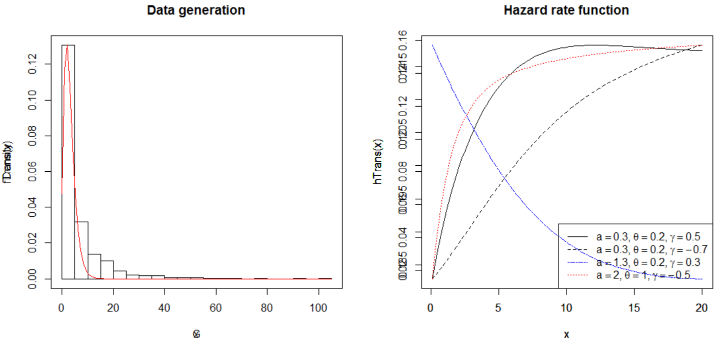


Figura: Data generation for $\gamma = -0.5, a = 0.3, \theta = 0.7$ and the hazard rate function

Maintenance model

- Considering the shock model proposed by Lam. Y (2007) [5], we developed a new shock model considering two types of shocks.
- A system can be affected by multiple shock that are independent with one producing more damage than the other one, so the time of repair is greater.
- Let suppose we have a system with one component. The random external deteriorating factors reduces the functioning time of the system

Definition

[5] A sequence of nonnegative random variables $\{X_n\}_{n\geq 0}$ is said to be a geometric process (GP) if they are independent and the distribution function of X_n is given by $F(a^{n-1}x)$ for $n \geq 0$, where $a > 0$ is called the ratio of the GP.

Assumptions

- We start with a newly instaled system at t=0. When the system fails it is repaired. A maintenance strategy N is adopted: the system is replaced with a new identical one after the Nth failure.
- Let X_n be the operating time of the system after the (n-1)th repair, $E(X_n) > 0$, $(X_n)_n$ form a geometric process with ratio $a \geq 1$.
- Let $Y_n = \alpha Y_n^{(1)} + (1-\alpha) Y_n^{(2)}$, $0 \leq \alpha \leq 1$ where $Y_n^{(1)}$ is the repair time of the system after the nth failure of type I, $\{Y_n^{(1)}\}_{n \geq 0}$ forms a geometric process with ratio $0 < b_1 \leq 1$, $E(Y_1^{(1)}) = \mu_1 \geq 0$, and $Y_n^{(2)}$ is the repair time of the system after the nth failure of type II, $\{Y_n^{(2)}\}_{n \geq 0}$ forms a geometric process with ratio $0 < b_2 \leq 1$, $E(Y_1^{(2)}) = \mu_2 \geq 0$, $\mu_1 \leq \mu_2$.
- Let Z be the replacement time, $E(Z) = \delta$

- The system suffers 2 kinds of shocks which occur randomly. Let $N_1(t)$ be the number of shocks of type I by time t, $(N_1(t))_t$ is a counting process having stationary and independent increment.
  Let $N_2(t)$ be the number of shocks of type II by time t, $(N_2(t))_t$ is a counting process having stationary and independent increment.
  Any shock arriving after failure gives no effect on the failed system. The succesive reductions in the system operating time are additive.
- Let $W_n^{(1)}$ be the reduction in the system operating time following the nth random shock of type I, $W_n^{(1)}$ are independent identically distributed.
  Let $W_n^{(2)}$ be the reduction in the system operating time following the nth random shock of type II, $W_n^{(2)}$ are independent identically distributed.
  $E(W_n^{(1)}) \leq E(W_n^{(2)})$

- $\{X_n\}_n$, $(N_1(t))_{t\geq 0}$, $(N_2(t))_{t\geq 0}$, $(W_n^{(1)})_n$ and $(W_n^{(2)})_n$ are independent
- Let r be the reward rate of the system and c the average cost of repair. The cost of replacement has two parts: the basic costs R and a part that is proportional with the time of replacement Z, with rate c_p representing the cost of labor, energy, etc.

- The number of shocks in $(t_{n-1}, t_{n-1}+t]$ is

$$N(t_{n-1}, t_{n-1}+t] = N(t_{n-1}+t) - N(t_{n-1})$$

where $N(t_{n-1})$ is the number of shocks until t_{n-1} time and $N(t_{n-1}+t)$ the number of shocks until $t_{n-1}+t$ time.

$$N(t_{n-1}) = N_1(t_{n-1}) + N_2(t_{n-1})$$

$$N(t_{n-1}+t) = N_1(t_{n-1}+t) + N_2(t_{n-1}+t)$$

where $N_1(t_{n-1})$ and $N_2(t_{n-1})$ are the number of shocks of type I, type II, respectively until t_{n-1} and $N_1(t_{n-1}+t)$ and $N_2(t_{n-1}+t)$ are the number of shocks of type I, type II, respectively until $t_{n-1}+t$.

- The total functioning reduced time in $(t_{n-1}, t_{n-1}+t]$ is

$$TR_{(t_{n-1},t_{n-1}+t]} = \sum_{i=N_1(t_{n-1})+1}^{N_1(t_{n-1}+t)} W_i^{(1)} + \sum_{i=N_2(t_{n-1})+1}^{N_2(t_{n-1}+t)} W_i^{(2)}$$
$$= \sum_{i=1}^{N_1(t_{n-1},t_{n-1}+t)} W_i^{(1)} + \sum_{i=1}^{N_2(t_{n-1},t_{n-1}+t)} W_i^{(2)}$$

The residual time at $t_{n-1} + t$ time is

$$S_n(t) = X_n - t - TR_{(t_{n-1}, t_{n-1}+t]}$$

If $S_n(t) < 0$, then the system fails.
The real functioning time of the system after the (n-1)th repair is given by

$$X_n^{'} = \inf_{t \geq 0} \{t | S_n(t) \leq 0\}$$

Lemma

$$P(X_n = t - TR_{(t_{n-1}, t_{n-1}+t]} > 0, \forall t \in [0, t^{'}])$$
$$= P(X_n - t^{'} - TR_{(t_{n-1}}, t_{n-1} + t^{'}] > 0)$$

For the real operating time of the system we have

$$P(X_n^{'} > t^{'} | N_1(t_{n-1}, t_{n-1} + t^{'}) = k_1, N_2(t_{n-1}, t_{n-1} + t^{'}) = k_2) \\ = 1 - \int_0^{\infty} F_n(t^{'} + x) dH_{k_1,k_2}(x)dx \qquad (3)$$

where $F_n(x) = F(a^{n-1}x)$ and $H_{k_1,k_2}(x)dx$ are the cumulative distribution function of X_n and $\sum_{i=1}^{k_1} W_i^{(1)} + \sum_{i=1}^{k_2} W_i^{(2)}$, respectively.

The real operating time of the system is

$$P(X_n' \leq x) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \int_0^{\infty} F_n(x+\omega) dH_{k_1,k_2}(\omega) \times P(N_1(x) = k_1) P(N_2(x) = k_2) \quad (4)$$

We say a cycle is complet when a replacement is complet. So, consecutive cycles together with the corresponding costs form a renewal reward process. [5].

Using the replacement strategy N: the system is replaced with a new identical one after the Nth failure, we obtain the following average cost:

$$C(N) = \frac{E\left[c\sum_{n=1}^{N-1} Y_n - r\sum_{n=1}^{N} X_n^{'} + R + c_p Z\right]}{E\left[\sum_{n=1}^{N-1} X_n^{'} + \sum_{n=1}^{N-1} Y_n + Z\right]}$$

$$= \frac{c\alpha\mu_1 \sum_{n=1}^{N-1} \frac{1}{b_1^{n-1}} + c(1-\alpha)\mu_2 \sum_{n=1}^{N-1} \frac{1}{b_2^{n-1}} - r\sum_{n=1}^{N} \lambda_n^{'} + R + c_p\delta}{\sum_{n=1}^{N} \lambda_n^{'} + \alpha\mu_1 \sum_{n=1}^{N-1} \frac{1}{b_1^{n-1}} + (1-\alpha)\mu_2 \sum_{n=1}^{N-1} \frac{1}{b_2^{n-1}} + \delta}$$

$$= P(N) - r$$

where $\lambda_n^{'} = E(X_n^{'})$ and

$$P(N) = \frac{(c+r)\left[\mu_1\alpha\sum_{n=1}^{N-1}\frac{1}{b_1^{n-1}} + \mu_2(1-\alpha)\sum_{n=1}^{N-1}\frac{1}{b_2^{n-1}}\right] + R + c_p\delta + r\delta}{\sum_{n=1}^{N}\lambda_n^{'} + \alpha\mu_1\sum_{n=1}^{N-1}\frac{1}{b_1^{n-1}} + (1-\alpha)\mu_2\sum_{n=1}^{N-1}\frac{1}{b_2^{n-1}} + \delta}$$

Let

$$h(N) = \frac{1}{(R + c_p\delta + r\delta)\Big[(b_1 b_2)^{N-1}\lambda'_{N+1} + \alpha\mu_1 b_2^{N-1} + \mu_2(1-\alpha)b_1^{N-1}\Big]}$$

$$\times \Bigg\{(c+r)\Big[(b_2^{N-1}\mu_1\alpha + \mu_2(1-\alpha)b_1^{N-1})\sum_{n=1}^{N}\lambda'_n$$

$$- \lambda'_{N+1}(b_2^{N-1}\mu_1\alpha\sum_{n=1}^{N-1}b_1^n + b_1^{N-1}\mu_2(1-\alpha)\sum_{n=1}^{N-1}b_2^n)$$

$$+ \delta(\mu_1\alpha b_2^{N-1} + \mu_2(1-\alpha)b_1^{N-1})\Big]\Bigg\}$$

We have the following results

Lemma

λ_n' is a nonincreasing function in n

Lemma

h(N) is a nondecreasing function in N

Theorem

The optim replacement strategy N^* is determined so that

$$N^* = min\{N|h(N) \geq 1\}$$

N^* is unique if and only if $h(N^*) > 1$.

- Considering the extended model proposed we consider two cases and for each of it simulation application.
We consider the reduction times in the system operating time to be exponential distributed, $W_n^{(1)} \sim Exp(\lambda_1)$ and $W_n^{(2)} \sim Exp(\lambda_2)$, $\lambda_1 \leq \lambda_2$ and 2 cases
  - **Case 1** $(X_n)_n \sim QGL(a_1, \theta_1)$ and $(Y_n^{(1)})_n \sim QGL(a_2, \theta_2), (Y_n^{(2)})_n \sim QGL(a_3, \theta_3)$
  - **Case 2** $(X_n)_n \sim TQGL(a_1, \theta_1, \gamma_1)$ and $(Y_n^{(1)})_n \sim TQGL(a_2, \theta_2, \gamma_2), (Y_n^{(2)})_n \sim TQGL(a_3, \theta_3, \gamma_3)$
- We consider also that $(N_1(t))_t$ and $(N_2(t))_t$ are Poisson processes of intensity λ_{N_1} and λ_{N_2}.

Theorem

[5](2014) Let $X_i \sim Gamma(\alpha_i, \beta_i)$, $\alpha_i, \beta_i > 0$, $i = \overline{1, n}$, *independent and* $S = X_1 + \cdots X_n$. *Let* $X_m \sim Gamma(\alpha_m, \beta_m)$ *denote the approximating gamma distribution of* S *associated with the moment matching method. We have*

$$E(X_m) = \alpha_m \beta_m, \quad E(S) = \sum_{i=1}^{n} \alpha_i \beta_i$$

$$Var(X_m) = \alpha_m \beta_m^2, \quad Var(S) = \sum_{i=1}^{n} \alpha_i \beta_i^2$$

where $\alpha_m = \dfrac{\mu^2}{\sum_{i=1}^{n} \alpha_i \beta_i^2}$, $\beta_m = \dfrac{\sum_{i=1}^{n} \alpha_i \beta_i^2}{\mu}$ *and* $\mu = \sum_{i=1}^{n} \alpha_i \beta_i$

The distribution of $\sum_{i=1}^{k_1} W_i^{(1)} + \sum_{i=1}^{k_2} W_i^{(2)}$ is given by the following theorem

Theorem

Let $S = \sum_{i=1}^{k_1} W_i^{(1)} + \sum_{i=1}^{k_2} W_i^{(2)}$. *Then*

$$S \sim Gamma(\alpha_m, \beta_m)$$

, *where*

$$\alpha_m = \frac{(k_1\lambda_1 + k_2\lambda_2)^2}{k_1\lambda_1^2 + k_2\lambda_2^2}, \quad \beta_m = \frac{k_1\lambda_1^2 + k_2\lambda_2^2}{k_1\lambda_1 + k_2\lambda_2}$$

Case 1

The probability function of the real functioning time of the system after the (n-1)th repair is

$$P(X_n' \leq x) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \left\{ 1 - \frac{e^{-\theta_1 x a^{n-1}}}{(a_1+1)\beta_m^{\alpha_m}\Gamma(\alpha_m)} I \right\} \times \frac{e^{-\lambda_{N_1}x}(\lambda_{N_1}x)^{k_1}}{k_1!} \frac{e^{-\lambda_{N_2}x}(\lambda_{N_2}x)^{k_2}}{k_2!}$$

where $I = \theta_1 a^{n-1} z^{\alpha_m+1}\Gamma(\alpha_m)\Psi(\alpha_m, \alpha_m+2, \lambda_\Psi z)$,
$\lambda_\Psi = \theta_1 a^{n-1} + \frac{1}{\beta_m}$, $z = \frac{a_1+1+\theta_1 x a^{n-1}}{\theta_1 a^{n-1}}$; $\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}dx$ is the gamma function and $\Psi(\cdot,\cdot,\cdot)$ Kummer function,
$\Psi(a,b,u) =$
$$\frac{1}{\Gamma(a)}\int_0^\infty t^{a-1}(1+t)^{b-a-1}e^{-ut}dt, \quad Re(a), Re(u) > 0, b \in \mathbb{C}$$

Case 1.1

- For $\lambda_1 = \lambda_2 = 1$, $\lambda_{N_1} = 3, \lambda_{N_2} = 0.5$, $\delta = 4, r = 5, c = 5, R = 10, c_p = 5, \alpha = 0.4, b_1 = 0.87, b_2 = 0.7, a = 2$ and
- $a_1 = 0.02, \theta_1 = 2, a_2 = 0.03, \theta_2 = 3, a_3 = 0.05, \theta_3 = 2$

we have

Tabela: Case 1.1

N	h(N)	C(N)	N	h(N)	C(N)
20	0.7854168	4.753316	27	1.055361	4.971424
21	0.8246262	4.816599	28	1.093465	4.97922
22	0.8638435	4.864317	29	1.131432	4.984915
23	0.9023237	4.900046	30	1.169358	4.989065
24	0.9406939	4.926618			
25	0.9790204	4.946283			
26	**1.017254**	**4.960777**			

Case 1.2

- For $\lambda_1 = \lambda_2 = 1$, $\lambda_{N_1} = 3, \lambda_{N_2} = 0.5$, $\delta = 4, r = 5, c = 5, R = 10, c_p = 5, \alpha = 0.4, b_1 = 0.87, b_2 = 0.7, a = 2$ and
- $a_1 = 1.7, \theta_1 = 4, a_2 = 3, \theta_2 = 7, a_3 = 4, \theta_3 = 2$ we have

Tabela: Case 1.2

N	h(N)	C(N)	N	h(N)	C(N)
20	0.7636672	4.613763	**27**	**1.02722**	**4.954912**
21	0.8018419	4.712099	28	1.063932	4.96722
22	0.8397631	4.786613	29	1.100854	4.97621
23	0.8773941	4.842579			
24	0.9153349	4.884313			
25	0.9527026	4.915274			
26	0.9897275	4.938122			

Case 1.2

- For $\lambda_1 = \lambda_2 = 1$, $\lambda_{N_1} = 3, \lambda_{N_2} = 0.5$, $\delta = 4, r = 5, c = 5, R = 10, c_p = 5, \alpha = 0.4, b_1 = 0.87, b_2 = 0.7, a = 2$ and
- $a_1 = 100, \theta_1 = 4, a_2 = 3000, \theta_2 = 7, a_3 = 4000, \theta_3 = 2$ we have

Tabela: Case 1.3

N	h(N)	C(N)	N	h(N)	C(N)
20	0.7593386	4.543052	**27**	**1.021854**	**4.946206**
21	0.7974994	4.658628	28	1.059124	4.960862
22	0.8353168	4.746551	29	1.096204	4.97158
23	0.8729748	4.812782			
24	0.9103821	4.862286			
25	0.947708	4.899051			
26	0.9848449	4.926218			

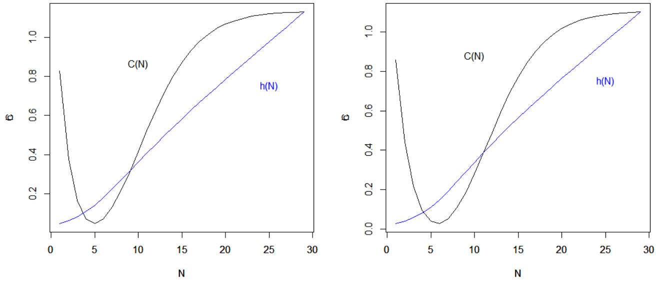


Figura: Case 1.1 and Case 1.2

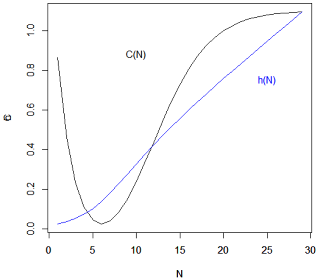


Figura: Case 1.3

Case 2

The probability function of the real functioning time of the system after the (n-1)th repair is

$$P(X_n' \leq x) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\left\{1 + \frac{e^{-\theta_1 x a^{n-1}}}{(a_1+1)\beta_m^{\alpha_m}\Gamma(\alpha_m)} I(\gamma-1) - \gamma J\right\} \times \frac{e^{-\lambda_{N_1}x}(\lambda_{N_1}x)^{k_1}}{k_1!}\frac{e^{-\lambda_{N_2}x}(\lambda_{N_2}x)^{k_2}}{k_2!}$$

where $I = \theta_1 a^{n-1} z^{\alpha_m+1}\Gamma(\alpha_m)\Psi(\alpha_m, \alpha_m+2, \lambda_\Psi z)$,
$\lambda_\Psi = \theta_1 a^{n-1} + \frac{1}{\beta_m}$, $z = \frac{a_1+1+\theta_1 x a^{n-1}}{\theta_1 a^{n-1}}$ and
$J = \frac{e^{-2\theta_1 x a^{n-1}}\theta_1^2 a^{2(n-1)}}{(a_1+1)^2\beta_m^{\alpha_m}\Gamma(\alpha_m)} z^{\alpha_m+2}\Gamma(\alpha_m)\Psi(\alpha_m, \alpha_m+3, \lambda_\Psi^* z)$,
$\lambda_\Psi^* = 2\theta_1 a^{n-1} + \frac{1}{\beta_m}$

Case 1.2

- For $\lambda_1 = \lambda_2 = 1$, $\lambda_{N_1} = 3, \lambda_{N_2} = 0.5$, $\delta = 4, r = 5, c = 5, R = 6, c_p = 3, \alpha = 0.7, b_1 = 0.3, b_2 = 0.8, a = 2$ and
- $a_1 = 0.5, \theta_1 = 8, \gamma_1 = -0.5, a_2 = 0.7, \theta_2 = 4, \gamma_2 = -0.6, a_3 = 0.4, \theta_3 = 10, \gamma_3 = -0.9$ we have

Tabela: Case 1.3

N	h(N)	C(N)
10	0.7887837	4.990034
11	0.851982	4.996723
12	0.9133332	4.998933
13	0.974314	4.999655
14	**1.033471**	**4.999889**
15	1.09242	4.999965
16	1.152533	4.999989

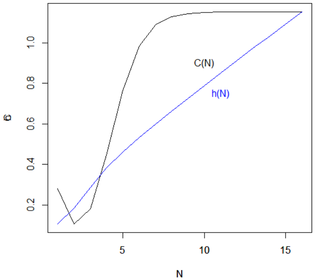


Figura: Case 2

Conclusions

- There isn't a huge difference between the quasi Lindley distribution and the exponential and gamma distribution in terms of the maintenance model
- Using the new model we can develop replacements strategies for systems that are subjected to multiple shocks that come at the same moment or not

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Thank you for you attention!